Benha University Faculty of Engineering- Shoubra Eng. Mathematics & Physics Department Qualifying Studies (Mathematics)		Final Term Exam Date: 26 / 5 / 2015 Operations Research EMM 406 Duration: 3 hours
• Answer **All** questions The exam consists of one page		• No. of questions: 4 Total Mark: 200

[1](a)Write the mathematical form of mathematical programming problem. Also, classify the mathematical programming problems.	30
(b)Write the dual problem of the LP problem: maximize $f = -2x + 2y$ s.t $3x + y \le 12$, $-x + y = 6$, $x + y \ge 8$, $x, y \ge 0$	30
[2]Solve the LP problems: (a) maximize $f = 3x + y + 4z$ s.t $x + y + 2z \le 18$ $2x + 3y + 2z = 18$ $x + 2y + 2z \ge 6$, $x, y, z \ge 0$	30
(b)minimize $f = -x + 3y - 3z$ s.t $2x + y - z \le 4$, $4x - 3y \le 2$, $-3x + 2y + z \le 3$, $x, y, z \ge 0$	30
[3](a) State the definition of convex set.	5
(b) State the definition of concave function.	5
(c)Prove that: (i)The minimum of a strict convex function f on a convex set $\mathbf{G} \subset \mathbf{R}^{\mathbf{n}}$ if exist must be unique.	20
(ii)The minimum of a non constant function f on a convex set $\mathbf{G} \subset \mathbf{R}^{\mathbf{n}}$ can not be attained at interior point.	20
[4]A company makes desk organizers. The standard model requires 2 hours of the cutter time and one hour of the finisher time. The deluxe model requires one hour of the cutter time and 2 hours of the finisher time. The cutter has 104 hours of time available for this work per month, while the finisher has 76 hours of time available for work. The standard model brings a profit of LE 5 per unit, while the deluxe model brings a profit of LE 9 per unit. The company wishes to make the most profit. How much of each model should be made in each month.	30

Good Luck

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Model Answer

Question 1

(a) A mathematical programming problem can be formulated as follows:

maximize (or minimize) $f(x)$

subject to $M = \{x \in R^n : g_r(x) \leq 0,\ r = 1,2,\dots,m\}$

f (x) is called the objective function

x is the vector of the variables (decision variables, unknowns)

$g_r(x) \leq 0,\ r = 1,2,\dots,m$ are constraints.

M is called the feasible domain of the problem which is formed by the constraints.

The mathematical programming problems can be classified as:

1- Linear programming (LP) problems when f(x) and $g_r(x) \leq 0,\ r = 1,2,\dots,m$ are linear functions. It take the form

Maximize (or minimize) f(x) = $c_1x_1 + c_2x_2 + \dots + c_nx_n$

subject to

$$\begin{aligned} & a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \leq b_1 \\ & a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n \leq b_2 \\ & \vdots \\ & a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n \leq b_m \\ & x_1, x_2, \dots, x_n \geq 0. \end{aligned}$$

This problem can be written in matrix form as follows:

Maximize (or minimize) f(x) = C x

s.t $Ax \leq B,\quad x \geq 0$

2- Non linear programming problems if f(x) is non linear or either one from $g_r(x) \leq 0,\ r = 1,2,\dots,m$ is non linear function.

3- Quadratic programming problems if f(x) is quadratic function and $g_r(x) \leq 0,\ r = 1,2,\dots,m$ are linear functions.

4- Integer programming problems if the decision variables are integers.

5- Mixed integer programming problems if some of the decision variables are integers.

--*30 Marks*

(b)The dual problem is:

Minimize $g(y) = 12y_1 + 6y_2 + 8y_3$

$$\begin{aligned} & 2y_1 - y_2 + y_3 \geq -2 \\ & y_1 + y_2 + y_3 \geq 2, \\ & y_1 \geq 0,\quad y_2 \text{ unrestricted},\quad -y_3 \geq 0 \end{aligned}$$

--*30 Marks*

Question 2

(a)The standard form of this problem is:

minimize $f = -x + 3y - 3z$

s.t $2x + y - z + s_1 = 4$

$4x - 3y + s_2 = 2$

$-3x + 2y + z + s_3 = 3, \quad x, y, z, s_1, s_2, s_3 \geq 0$

where s_1, s_2 and s_3 are slack variables.

The steps of the simplex method goes as:

B.V	x	y	z	s_1	s_2	s_3	Solu
s_1	2	1	-1	1	0	0	4
s_2	4	-3	0	0	1	0	2
s_3	-3	2	1	0	0	1	3
f	1	–3	3	0	0	0	0
s_1	-1	3	0	1	0	1	7
s_2	4	-3	0	0	1	0	2
z	-3	2	1	0	0	1	3
f	10	–9	0	0	0	–3	–9
s_1	0	9/4	0	1	1/4	1	15/2
x	1	–3/4	0	0	1/4	0	1/2
z	0	–1/4	1	0	3/4	1	9/2
f	0	–3/2	0	0	–5/2	–3	–14

This is the optimum case. Then, the optimal solution is:

$(x^*, y^*, z^*) = (1/2, 0, 9/2)$ and the optimal value is $f^* = -14$.

--*30 Marks*

(b) The standard form of this problem is:

maximize $f = 3x + y + 4z$

s.t $x + y + 2z + s_1 = 18$

$2x + 3y + 2z + u = 18$

$x + 2y + 2z - t + v = 6, \quad x, y, z, s_1, t, u, v \geq 0$

where s_1 is slack variable, t is surplus variable and u, v are artificial variables.

Let w = u + v. Then, the objective of phase one is:

$w + 3x + 5y + 4z - t = 24$

The steps of phase one goes as table :

B.V	x	y	z	t	s_1	u	v	Solu
s_1	1	1	2	0	1	0	0	18
u	2	3	2	0	0	1	0	18
v	1	2	2	-1	0	0	1	6
f	-3	-1	-4	0	0	0	0	0
w	3	5	4	-1	0	0	0	24
s_1	1/2	0	1	1/2	1	0	–1/2	15
u	1/2	0	-1	3/2	0	1	–3/2	9
y	1/2	1	1	–1/2	0	0	1/2	3
f	–3/2	0	-3	–1/2	0	0	1/2	3
w	1/2	0	-1	3/2	0	0	–5/2	9
s_1	1/3	0	4/3	0	1	–1/3	0	12
t	1/3	0		1	0	2/3	-1	6
y	– 2/3			0	0	1/3	0	6
	2/3	1	2/3					
f	–4/3	0	–10/3	0	0	1/3	0	6
w	0	0	0	0	0	-1	-1	0

This is the end of phase one.
Phase two starts with the following table which is formed by deleting the columns of r, v and the row of w.

B.V	x	y	z	t	s_1	Solu
s_1	1/3	0	4/3	0	1	12
t	1/3	0	–2/3	1	0	6
y	2/3	1	2/3	0	0	6
f	–4/3	0	–10/3	0	0	6
s_1	-1	-2	0	0	1	0
t	1	0	0	1	0	12
z	1	3/2	1	0	0	9
f	2	5	0	0	0	36

This is the optimum case. Then, the optimal solution is:
(x*, y*, z*) = (0, 0, 9) with optimal value f * = 36.

--*30 Marks*

Question 3

(a)**Convex set**: A set M is called convex if for all two points x and y in M, all points of the line segment $\lambda x + (1 - \lambda)y,\ \ 0 < \lambda < 1$, lie in M.

---*5 Marks*

(b)**Concave function**: A function f is called concave on a set M if for all two points x and y in M, $f(\lambda x + (1-\lambda)y) \geq \lambda f(x) + (1-\lambda)f(y), \ \ 0 < \lambda < 1$

---*5 Marks*

(c)Theorems:

(i)The minimum of a strict convex function f on a convex set $\mathbf{G} \subset \mathbf{R}^{\mathbf{n}}$ if exist must be unique.

---*20 Marks*

(ii)The minimum of a non constant function f on a convex set $\mathbf{G} \subset \mathbf{R}^{\mathbf{n}}$ can not be attained at interior point.

---*20 Marks*

Question 4

Let x be the number of standard model and y be the number of deluxe model.

Then, the objective function is: f = 6x + 11y

	x	y	available
cutter time	2	1	104
finisher time	1	2	76

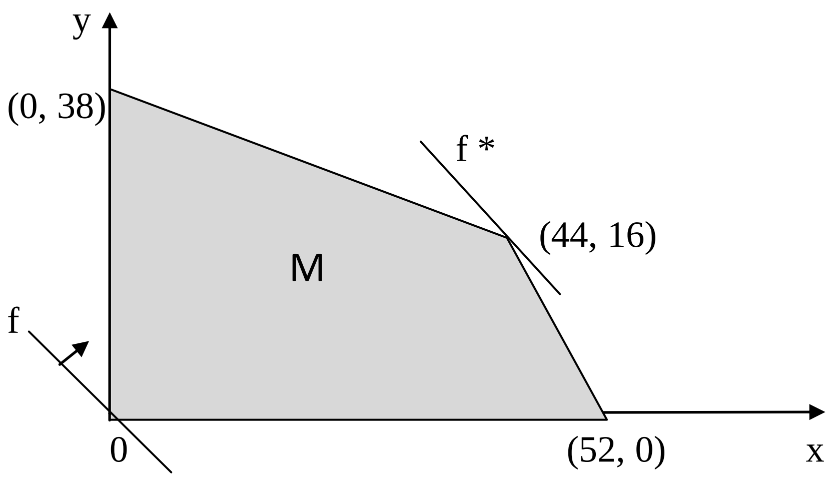
The constraints are: $x + 2y \leq 76$, $2x + y \leq 104$.

Then, the LP model is: maximize $f = 6x + 11y$

s.t $x + 2y \leq 76$

$2x + y \leq 104, \ \ x, y \geq 0$

This LP problem can be solved graphically as shown in the Figure.



The maximum profit is LE 440 at the optimal solution (44,16).

---*30 Marks*

Dr. Mohamed Eid